MODELING AND CONTROLLER DESIGN FOR AN ACTIVE CAR SUSPENSION SYSTEM USING HALF CAR MODEL

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MODELING AND CONTROLLER DESIGN FOR AN ACTIVE CAR SUSPENSION SYSTEM USING HALF CAR MODEL

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A project report submitted in part of fulfillment of the requirements for the award of the Degree of the Bachelor of Engineering (Electrical – Control and Instrumentation)

Fakulti Kejuruteraan Elektrik
Universiti Teknologi Malaysia

MAY 2009
DECLARATION

I declare that this project report entitled “Modeling and Controller Design for an Active Car Suspension System Using Half Car Model” is the result of my own research except as cited in the references. The project report has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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Specially..

To my beloved parents
To my kind brothers and sisters
And not forgetting to all friends
For their
Love, Sacrifice, Encouragements, and Best Wishes
ACKNOWLEDGEMENTS

Praise be to Allah S.W.T to Whom we seek help and guidance and under His benevolence we exist and without His help this project could not have been accomplished.

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ABSTRACT

The concept of using an active suspension system for vehicles is to provide the best performance of car controlling. A fully active suspension system aim is to control the suspension over the full bandwidth of the system. It is considered to be the way of increasing load carrying, handling and ride quality.

This project compares the passive suspension response with active suspension response. The response of the system is simulated by MATLAB software. In order to make the MATLAB software, the SIMULINK in MATLAB is used to write a program for the designed controller.

This project presents brief for the different types of suspension system such as passive, semi-active and active suspension system. The physical and mathematics modeling for a half car suspension system are presented. The derivations of the mathematical modeling for the passive and active suspension are presented in state space form.

Besides that, the controller design methods using MATLAB are also presented. In order to have a complete idea about the function and the using method of the software simulation program by SIMULINK, the instruction of the whole program are provided. The end result of this software is analyzed and discussed to prove the controlling effect of the active suspension system.
ABSTRAK

Sistem suspensi aktif digunakan pada kenderaan bagi memberi kecekapan terbaik pengawalan kereta. Sistem suspensi yang aktif bertujuan mengawal suspensi untuk system berlebar jalur penuh. Ia menjadi cara untuk meningkatkan jumlah pengangkutan beban, pengawalan kenderaan dan juga keselesaan pengguna.

Projek ini membandingkan sambutan suspensi pasif dengan sambutan aktif. Sambutan sistem itu adalah dihasilkan oleh perisian MATLAB. Bagi menghasilkan perisian simulasi MATLAB, aplikasi SIMULINK di dalam MATLAB digunakan bagi menulisaturcara bagi tujuan rekabentuk pengawal.

Projek ini membentangkan secara ringkas jenis-jenis sistem suspensi yang berbeza seperti suspensi pasif, separuh aktif dan sistem suspensi aktif. Rupabentuk fizikal dan perwakilan matematik untuk sistem suspensi separuh kereta dibincangkan. Terbitan persamaan matematik bagi sistem suspensi aktif dan pasif dipersembahkan di dalam bentuk matrik.

Selain itu, pengawal direka bentuk dengan menggunakan kaedah MATLAB ada dipersembahkan di dalam kajian ini. Bagi meningkatkan tahap pemahaman, kesuluruhan idea lengkap mengenai fungsi dan kaedah bagi simulasi perisian program oleh SIMULINK, aturcara program keseluhan projek juga dibincangkan. Keputusan akhir perisian simulasi ini dianalisa demi membuktikan kecekapan sistem suspensi aktif.
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CHAPTER 1

INTRODUCTION

1.1 Introduction

Suspension system, system that connects the wheels of the automobile to the body, in such a way that the body is cushioned from jolts resulting from driving on uneven road surfaces. The suspension affects an automobile’s comfort, performance, and safety.

The suspension system suspends the automobile’s body a short distance above the ground and maintains the body at relatively constant height to prevent it from pitching and swaying. In order to maintain effective acceleration, braking, and cornering the components of good handling, the suspension system must also keep all four tyres firmly in contact with the ground.

The automotive suspension system is designed to compromise between the comfort as the road handling can be improved by using the electronically controlled suspension system. Hence, the suspension system may be categorized as passive, semi-active or active suspension systems.
1.2 Suspension system types

Basically there are three types of car suspension system; passive, semi-active and active suspension system.

1.2.1 Passive suspension system

A passive suspension system includes the conventional springs and shock absorbers. Such system has an ability to store energy via spring and to dissipate in via damper. In order to achieve a certain level of compromise between road holding, load carrying and comfort, its parameters are generally fixed. Figure 1.1 shows the diagram of passive suspension system.

![Diagram of Passive Suspension System](image)

**Figure 1.1:** Passive Suspension System.
1.2.2 Semi-Active Suspension System

A semi-active suspension system provides controlled real-time dissipation of energy. A mechanical device called active damper is fixed in parallel with a conventional spring. It does not provide any energy to the system. Figure 1.2 shows the semi-active suspension system.

![Semi-Active Suspension System Diagram](image)

**Figure 1.2:** Semi Active Suspension System
1.2.3 Active Suspension System

Active suspension system has an ability to store, dissipate and to introduce energy to the system. The hydraulic actuator is connected in parallel with a spring and absorber. While, sensor of the body are located at different points of the vehicle to measure the motions of the body. It may vary its parameters depending upon operating conditions. Figure 1.3 shows the active car suspension system.

![Active Suspension System Diagram](image_url)

Figure 1.3: Active Suspension system.
1.3 Active suspension System Overview

In a fully active suspension system, generally an actuator connected between the sprung and unsprung masses of the vehicle. As shown in Figure 1.3, the systems consist of hydraulic actuator mounted between car body and car wheel. The actuator can generate control forces (calculated by a computer) to suppress the system responds to the changes of the road condition, thus holding the vehicle in a constant state of “equilibrium.”

The components of the active car suspension are illustrated in Figure 1.4:

![Figure 1.4: Component of Active Car Suspension.](image)

There are four different actuators, one at each wheel. The active suspensions are equipped with sensors, which are linked to a powerful computer system, which has information about the vehicle and its response to different road conditions.

The system process information obtained from the sensors and then sends a signal to provide and appropriate response in the actuator. The computer system and actuator will keep the car level on a smooth surface. However if the vehicle were to encounter an irregularity in the road surface or a bend, then the signals from the sensors will enable the computer system to calculate the change in load in that particular actuator and cause response to compensate for the change in load.

Many people are not aware of any change in road conditions, since the time for the sensors to detect the change and the actuator to respond is a matter of
milliseconds. The active suspension system must support the car, provide directional control during car handling and provide effective isolation of passengers or payload from road disturbances.

The project is subject to design the controller for an active car suspension system using Linear Quadratic Regulator controller approach. The controller must be able to control the relative motion between car body and wheels. Besides, the fully feedback controller are also aimed to control the car body acceleration for smooth riding and tyre deflection in order to improve the road handling.

In order a clearer insight into the performance of the different controller, quantitative comparison between the different simulation result from two controllers and passive system will be carried out. The performance will be simulated by using MATLAB software. Moreover the parameters depending upon operating condition.

1.4 Objective of Project

The core of this project is to design LQR controller for a suspension system using half car model. Before starting to develop the software to simulate this system by using MATLAB, it is important to analyze and understand active suspension system for half car model. After the data obtained from the simulation, the result will be used to analyze and justify the best parameter for the controller.

1.5 Scope of the Project

This project focused on the design of half car’s active suspension system. The controller will be designed using LQR controller method. The MATLAB software is used to simulate the response.
After the results obtained from both of the method, the result will be compared to conclude which controller perform well for this kind of system.

1.6 Outline of Thesis

This thesis consists of five chapters. In chapter 1, it discuss about the objective and scope of this project as long as summary of works. The primary goal to this chapter is to provide the reader with a rough idea about this thesis.

While Chapter 2 will discuss more on theory and literature reviews that have been done. It well discuss about types of suspension, various kind of experiment that already being done related to active car suspension system using appropriate controller to give comfort ride to the driver and passengers. Besides that, the mathematical model of the system is also presented on this chapter.

Chapter 3 deals with the modeling of a half car suspension system. Firstly, the state space representation of the dynamic model of passive suspension for a half car models are outlined. Secondly, the state space representation of the dynamic model of the passive suspension for a half car model is outlined. Then the state representation of the dynamic model of the active suspension with force input for a half car model is composed. Finally, road profile that represents the disturbance in the suspension will be represented.

Chapter 4 cover the theory of Optimal Control as well as the design procedure of the Optimal Controller in the suspension system.

The software development of the system is presented in chapter 4. This includes the SIMULINK block diagram and the interfacing in running the system in real-time controlling mode.

Chapter 5 discusses the performance of the LQR Controller. All the results are presented in graphical form to ease the process of data analysis. This chapter also includes the comparisons between these two controllers.
Finally, chapter 6 the last chapter conclude the results of the thesis. It also discusses the problem encountered during the whole course of completing this project and some useful suggestions for future research of this project.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The suspension system is a nonlinear and unstable system, thus providing a challenge to the control engineers or researchers. It has become a benchmark to improve ride comfort and road handling. There are many efforts that have been done to develop the controller for this system.

2.2 Literature Research

2.2.1 State Space Controller

In year 1997, Carneige Mellon [3] from University of Michigan presented the procedure of designing state space controller in MATLAB for bus suspension system. It was implemented by using the pole placement method. The proposed method was applied to a suspension system, and simulation results show that substantial improvement in the performance was achieved compared with other local observers.
The study conducted by Yahya Md. Sam[22][23] shows that an active suspension gives a better performance in terms of comfort ride compared to the passive suspension. An active suspension also increases a tire-to-road contact in order to make the vehicle more stable. This concludes that LQR controller can be considered one of the solutions for excellent comfort ride and good handling of car in the new millennium.

### 2.2.2 Fuzzy Logic Controller

In 1999, Yoshimura [25] presented an active suspension system for passenger cars using linear and fuzzy logic control technique. The study utilize vertical acceleration of the vehicle body as the vertical acceleration of the vehicle body as the principle source of control, and the fuzzy logic control scheme as the complementary control of the active suspension system for passenger cars. The fuzzy control rules are determined by minimizing the mean square of the time responses of the vehicle body under certain constraints on the acceptable relative displacements between vehicle body and suspension parts and tire deflections. The simulation results showed the linear-fuzzy logic controls are effective in the vibration isolation of the vehicle body.

Shiqg-Jen Wu [19], describes a fuzzy based intelligent active suspension system does provide improved ride quality by minimizing both the displacement and accelerations of vehicle center and pitch angle at the same time.

### 2.2.3 Others Controller Approaches

There are few more controller that have been developed for active suspension system such as, hydraulic controlled actuator and Neural-Fuzzy Controller approaches.

To counter the robustness issues Yahaya [22][24] examined the application of Proportional Integral Sliding Mode Control (PISMC) in order to improve the ride
comfort and road handling. The additional integral in the proposed sliding surface provides one more degree of freedom and also reduce the steady state error. A computer simulation performed to demonstrate the effectiveness and robustness of the proposed control scheme. The result shows that the use of the proposed proportional integral sliding made control technique proved to be effective in controlling the vehicle and is more robust as compared to the linear quadratic regulator method and the passive suspension system.

In order to simulate a real active suspension controller, Yahaya [22] further his studies by considering the hydraulic dynamic. Modeling of the active suspension systems in the early days considered that the input to the active suspension is a linear force. Recently, due to the development of new control theories, the force input to the active suspension systems has been replaced by an input to control the actuator. Therefore, the dynamics of the active suspension systems now consists of the dynamics of suspension system plus the dynamic of the actuator systems. Hydraulic actuators are widely used in the active suspension system. The studies shows that hydraulic dynamics that is based on variable structure control theory, which is capable of satisfying all the pre-assigned design requirements within the actuators limitation.

Shinq Jen Wu [19] study simulation and analysis on the developing the advanced design and synthesis of self-learning optimal intelligent active suspension systems. Artificial neural-based fuzzy modeling is applied to set up the neural-based fuzzy model based on the training data from the nonlinear half car suspension system dynamics. The development of self-learning optimal intelligent active suspension can not only absorb disturbance and shock, to adapt the model, the sensor and the actuator error but also cope with the parameter uncertainty with minimum power consumption.
2.3 Conclusion

This chapter discuss about works that have been done by the researchers in suspension system. There are a number of controllers which has been studied for years that can be used to increase ride comfort and road handling for suspension system. This thesis wills emphasis to the several efforts that utilize the state space designed for linear model. Meanwhile, LQR controller is proposed for nonlinear model with disturbance effect. This method proposed in this study in order to control the actuator to have satisfactory road holding ability, while still providing comfort when riding over bumps and holes in the road.
CHAPTER 3

SYSTEM MODEL

3.1 Introduction

A suspension system purpose is to give both comfort ride and road handling for passengers inside the car. The development of an active suspension system for a vehicle is of a great interest in both academic and industrial fields. The study of active suspension system has been performed using various suspension models. Generally, a vehicle suspension models are divided into three types: a quarter car, a half car and a full car models.

In the quarter car model, the model takes into account the interaction between the quarter car body and the single direction. For the half car model, the interactions are between the car body and the wheels and also between both ends of the car body. The first interaction in the half car model caused the vertical motion and the second interaction produced an angular motion. In the full car model, the interactions are between the car body and the four wheels that generate the vertical motion, between the car body and the left and right wheels that generate an angular motion called rolling and between the car body and the front and rear wheels that produce pitch motion.

Half car model suspension system with a linear force input was used in this study. The linear force input is connected between the car body and the wheel. It may vary its parameters depending upon operating condition.
3.2 Suspension System for Half Car Model

In this section, a complete mathematical model of the passive suspension for a half car model, as shown in Figure 3.1 are derived based on the approach as presented in Yoshimura [25]. The passive suspension for the half car model consist of the front and rear wheels and also the axles that are connected to the half portion of the car body through the passive-springs-dampers combination, while each tire is modeled as a simple spring without damper. The parameter of the passive suspension components is used in the study is as presented in Yahaya [24] and is tabulated in Table 3.1. It is assumed that all springs and damper are linear.

![Figure 3.1: A Half Car Model](image)

The motion equation for the passive suspension for the half car model may be derived as follows (Yoshimura [25]):
\( m_b \ddot{x}_b + c_{bf}(\dot{x}_{bf} - \dot{x}_{w_f}) + k_{bf}(x_{bf} - x_{w_f}) + c_{br}(\dot{x}_{br} - \dot{x}_{w_r}) + k_{br}(x_{br} - x_{w_r}) = 0 \)

(3.1)

\( I_{bh} \ddot{\theta}_b + L_f [c_{hf}(x_{hf} - x_{w_f})] + k_{hf}(x_{hf} - x_{w_f}) - L_r [c_{hr}(x_{hr} - x_{w_r}) + k_{hr}(x_{hr} - x_{w_r})] = 0 \)

(3.2)

\( m_{w_f} \ddot{x}_{w_f} - c_{bf}(\dot{x}_{bf} - \dot{x}_{w_f}) - k_{bf}(x_{bf} - x_{w_f}) - k_{w_f}(x_{w_f} - w_f) = 0 \)

(3.3)

\( m_{w_r} \ddot{x}_{w_r} - c_{br}(\dot{x}_{br} - \dot{x}_{w_r}) - k_{br}(x_{br} - x_{w_r}) - k_{w_r}(x_{w_r} - w_r) = 0 \)

(3.4)

Where

\( m_b = \) mass of the car body (kg).

\( I_b = \) moment of inertia for the car body (kgm²).

\( \theta_b = \) rotary angle of the car body at the center of gravity (rad).

\( m_{w_f} = \) mass of the front wheel (kg).

\( m_{w_r} = \) mass of the rear wheel (kg).

\( k_{bf} = \) stiffness of the front car body spring (N/m).

\( k_{br} = \) stiffness of the rear car body spring (N/m).
\[ k_{tf} = \text{stiffness of the front car tire (N/m)}. \]

\[ k_{tr} = \text{stiffness of the rear car tire (N/m)}. \]

\[ c_{2f} = \text{damping of the front car damper (Ns/m)}. \]

\[ c_{2r} = \text{damping of the rear car damper (Ns/m)}. \]

\[ x_c = \text{vertical displacement of the car body at the center of gravity (m)}. \]

\[ x_{2f} = \text{vertical displacement of the car body at the front location (m)}. \]

\[ x_{2r} = \text{vertical displacement of the car body at the rear location (m)}. \]

\[ x_{wf} = \text{vertical displacement of the car wheel at the front wheel (m)}. \]

\[ x_{wr} = \text{vertical displacement of the car wheel at the rear wheel (m)}. \]

\[ w_f = \text{an irregular excitation from the road surface at the front car (m)}. \]

\[ w_r = \text{an irregular excitation from the road surface at the rear car (m)}. \]

\[ L_f = \text{distance of the front suspension location with reference to the center of gravity of the car body (m)}. \]

\[ L_r = \text{distance of the rear suspension location with reference to the center of gravity of the car body (m)}. \]

\[ L = \text{total distance of the front suspension to the rear suspension, i.e } L_f + L_r \text{ (m)}. \]

in order to consider the vertical displacement of the front and rear car body, \( x_{bf} \) and \( x_{br} \) respectively, the constraints as presented in Yohimura [25] are used to
replace the vertical displacement of the car body at the center of gravity, \(x_b\), and the rotary angle of the car body at the center gravity, \(\theta_b\). The constraints are:

\[
x_b = \left( L_f x_{bf} + L_r x_{br} \right) / L \tag{3.5}
\]

\[
\theta_b = (x_{bf} - x_{br}) / L \tag{3.6}
\]

Therefore equations (2.1) and (2.2) can be rewritten as,

\[
\frac{m_b}{L} \left( L_f \ddot{x}_{bf} + L_r \ddot{x}_{br} \right) + c_{bf} (\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf} (x_{bf} - x_{wf}) + c_{br} (\dot{x}_{br} - \dot{x}_{wr}) + k_{br} (x_{br} - x_{wr}) = 0 \tag{3.7}
\]

\[
\frac{I_b}{L} (\ddot{x}_{bf} - \ddot{x}_{br}) + L_f \left[ c_{bf} (\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf} (x_{bf} - x_{wf}) \right] - L_r \left[ c_{br} (\dot{x}_{br} - \dot{x}_{wr}) + k_{br} (x_{br} - x_{wr}) \right] = 0 \tag{3.8}
\]

Equations (2.3), (2.4),(2.7) and (2.8) can be written in the following form:

\[
M_{hm} \ddot{X}_{hm} (t) + S_{hm} \dot{X}_{hm} (t) + T_{hm} X_{hm} (t) = E_{hm} W_{hm} (t) \tag{3.9}
\]

Where

\[
X_{hm} (t) = [\dot{x}_{bf}(t) \ x_{bf}(t) \ x_{br}(t) \ x_{wr}(t)]^T \tag{3.10}
\]

\[
W_{hm} (t) = [w_f(t) \ w_r(t)]^T; \tag{3.11}
\]

\[
M_{hm} = \\
\begin{bmatrix}
\frac{L_f m_b}{L} & 0 & \frac{L_r m_b}{L} & 0 \\
\frac{I_b}{L} & 0 & -\frac{I_b}{L} & 0 \\
0 & m_{wf} & 0 & 0 \\
0 & 0 & 0 & m_{wr}
\end{bmatrix} \tag{3.12}
\]
The state space representation of the motion equations may be written in the following form:

\[ \dot{x}_{h \dot{}}(t) = A_{h \dot{}} x_{h \dot{}}(t) + G_{h \dot{}} w_{h \dot{}}(t) \]  \hspace{1cm} (3.16)

Where

\[ x_{h \dot{}}(t) = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]^T, \]
\[ \dot{x}_{h \dot{}}(t) = [\dot{x}_{bf} \ \dot{x}_{bf} \ \dot{x}_{br} \ \dot{x}_{bf} \ \dot{x}_{wff} \ \dot{x}_{br} \ \dot{x}_{wfr}]^T \]  \hspace{1cm} (3.17)

\[ A_{h \dot{}} = \begin{bmatrix}
    a_{h11} & a_{h12} & a_{h13} & a_{h14} & a_{h15} & a_{h16} & a_{h17} & a_{h18} \\
    a_{h21} & a_{h22} & a_{h23} & 0 & 0 & a_{h26} & 0 & 0 \\
    a_{h31} & a_{h32} & a_{h33} & a_{h34} & a_{h35} & a_{h36} & a_{h37} & a_{h38} \\
    0 & 0 & a_{h43} & a_{h44} & 0 & 0 & a_{h47} & a_{h48} \\
    1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix} \]  \hspace{1cm} (3.18)
Where the non-zero elements of $A_{\mathcal{R}_i}$ and $G_{\mathcal{R}_i}$ matrices are:

$$\alpha_{h11} = \frac{-c_{bf}}{m_b} - \frac{L_f^2 c_{bf}}{I_2}, \quad \alpha_{h12} = \frac{c_{bf}}{m_b} + \frac{L_f^2 c_{bf}}{I_2}$$

$$\alpha_{h13} = \frac{-c_{br}}{m_b} - \frac{L_f L_r c_{br}}{I_2}, \quad \alpha_{h14} = \frac{c_{br}}{m_b} - \frac{L_f L_r c_{br}}{I_2}$$

$$\alpha_{h15} = \frac{-k_{bf}}{m_b} - \frac{L_f^2 k_{bf}}{I_2}, \quad \alpha_{h16} = \frac{k_{bf}}{m_b} + \frac{L_f^2 k_{bf}}{I_2}$$

$$\alpha_{h21} = \frac{-k_{br}}{m_b} - \frac{L_f^2 k_{br}}{I_2}, \quad \alpha_{h22} = \frac{k_{br}}{m_b} + \frac{L_f^2 k_{br}}{I_2}$$

$$\alpha_{h23} = \frac{c_{bf}}{m_{bf}}, \quad \alpha_{h24} = \frac{-c_{bf}}{m_{bf}}, \quad \alpha_{h25} = \frac{k_{bf}}{m_{bf}}, \quad \alpha_{h26} = \frac{-c_{bf}}{m_{bf}}$$

$$\alpha_{h31} = \frac{-c_{bf}}{m_b} + \frac{L_f L_r c_{bf}}{I_2}, \quad \alpha_{h32} = \frac{c_{bf}}{m_b} - \frac{L_f L_r c_{bf}}{I_2}$$

$$\alpha_{h33} = \frac{-c_{br}}{m_b} - \frac{L_f^2 c_{br}}{I_2}, \quad \alpha_{h34} = \frac{c_{br}}{m_b} + \frac{L_f^2 c_{br}}{I_2}$$
The performance of the passive suspension system for a quarter car model subjected to the parameters if the spring and damper. For the half car model, it can be observed that equation (2.18) has similar characteristics with equation with the quarter car model if the distance of the front and rear suspension from the center of gravity of the car body in the half car model is ignored. Therefore, the performance of a passive suspension system for the half car model is also purely determined by the springs and dampers that are all fixed. Similar to quarter car model, an actuator is added to the front and rear suspensions for the half car model that will transform the suspension type from passive to active.

\[
\begin{align*}
\alpha_{h88} &= \frac{-k_{bf}}{m_b} + \frac{L_f g R_{bf}}{I_b}, & \alpha_{h86} &= \frac{k_{bf}}{m_b} - \frac{L_f g R_{bf}}{I_b}, \\
\alpha_{h87} &= \frac{-k_{br}}{m_b} - \frac{L_f g R_{br}}{I_b}, & \alpha_{h88} &= \frac{k_{cr}}{m_b} + \frac{L_f g R_{cr}}{I_b}, \\
\end{align*}
\]

\[
\begin{align*}
\alpha_{h21} &= \frac{c_{fr}}{m_{fr}}, & \alpha_{h44} &= \frac{c_{fr}}{m_{fr}}, & \alpha_{h47} &= \frac{k_{bf}}{m_{fr}}, & \alpha_{h48} &= \frac{-L_f g R_{bf} R_{fr}}{m_{fr}}.
\end{align*}
\]

\[
\begin{align*}
\mathcal{G}_{h21} &= \frac{k_{bf}}{m_{fr}}, & \mathcal{G}_{h21} &= \frac{k_{cr}}{m_{cr}}.
\end{align*}
\]

(3.20)

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the car body, $m_b$</td>
<td>1794.4 Kg</td>
</tr>
<tr>
<td>Moment of the inertia for the car body, $I_b$</td>
<td>3443.05 Kgm2</td>
</tr>
<tr>
<td>Mass of the front wheel, $m_{fr}$</td>
<td>87.15 Kg</td>
</tr>
<tr>
<td>Mass of the rear wheel, $m_{cr}$</td>
<td>140.14 Kg</td>
</tr>
<tr>
<td>Stiffness of the front car body spring, $k_{bf}$</td>
<td>66824.2 N/m</td>
</tr>
<tr>
<td>Stiffness of the rear car body spring, $k_{br}$</td>
<td>18615 N/m</td>
</tr>
<tr>
<td>Stiffness of the front car tire, $k_{of}$</td>
<td>1011115 N/m</td>
</tr>
</tbody>
</table>
Stiffness of the rear car tire, $k_{wr}$ : 1011115 N/m
Damping of the front car damper, $c_{bf}$ : 1190 Ns/m
Damping of the rear car damper, $c_{br}$ : 1000 Ns/m

<table>
<thead>
<tr>
<th>Table 3.1: Parameter values for the half car suspension[24]</th>
</tr>
</thead>
</table>

The active suspension system of the half car model is shown in figure 2.4. Let $f_f$ and $f_r$ be the force inputs for the front and rear actuators, respectively. Therefore, the motion equations of the active suspension for the half car model may be determined as follows [25]:

\[
\frac{m_b}{L} \left( L_x \ddot{x}_{bf} + L_x \dot{x}_{br} + c_{bf} (x_{bf} - x_{wf}) + k_{bf} (x_{bf} - x_{wf}) + c_{bf} (x_{br} - x_{wr}) - k_{br} (x_{br} - x_{wr}) - f_f - f_r - \dot{u} \right) = 0
\]  

(3.21)

\[
\frac{I_b}{L} (\ddot{x}_{bf} - \dot{x}_{br}) + L_f [c_{bf} (x_{bf} - x_{wf}) + k_{bf} (x_{bf} - x_{wf}) - f_f] - L_r [c_{br} (x_{br} - x_{wr}) + k_{br} (x_{br} - x_{wr}) - f_f] = 0
\]  

(3.22)

\[
m_{wf} \ddot{x}_{wf} - c_{bf} (x_{bf} - x_{wf}) - k_{bf} (x_{bf} - x_{wf}) + k_{wf} (x_{wf} - \dot{u}) = 0
\]  

(3.23)

\[
m_{wr} \ddot{x}_{wr} - c_{br} (x_{br} - x_{wr}) - k_{br} (x_{br} - x_{wr}) + k_{wr} (x_{wr} - \dot{u}) = 0
\]  

(3.24)

Equations (2.21)-(2.24) can be written in the following form:

\[
M_{hi} \dot{X}_{hi}(t) + S_{hi} \ddot{X}_{hi}(t) + T_{hi} X_{hi}(t) = D_{hi} \dot{F}_i(t) + E_{hi} W_{hi}(t)
\]  

(3.25)

Where $X_{hi}(t)$, $W_{hi}(t)$, $M_{hi}$, $S_{hi}$, $T_{hi}$ and $E_{hi}$ are given by the equations (3.10)-(3.15) with the subscript $h_m$ being changed to $h_i$, while
Hence, the state space representation of the active suspension of the half car model may be obtained as:

\[ \dot{x}_{h_{la}}(t) = A_{h_{la}} x_{h_{la}}(t) + B_{h_{la}} u_{h_{la}}(t) + G_{h_{la}} w_{h_{la}}(t) \]  
\[(3.28)\]

Where \( x_{h_{la}}(t) \), \( w_{h_{la}}(t) \), \( A_{h_{la}} \), and \( G_{h_{la}} \) are given by equations (2.17)-(2.19) with the subscript \( h_{la} \) replacing \( h \), in those equation, while

\[ u_{h_{la}}(t) = F_h(t) = [f_f \ f_r]^T \]  
\[(3.29)\]

The input matrix, \( B_{h_{la}} \), is given as

\[
B_{h_{la}} = \begin{bmatrix}
  b_{11} & b_{12} \\
  b_{21} & 0 \\
  b_{31} & b_{32} \\
  0 & b_{42} \\
  0 & 0 \\
  0 & 0 \\
  0 & 0
\end{bmatrix}
\]  
\[(3.30)\]

Where the non-zero elements of \( B_{h_{la}} \) matrix are as follows:

\[
b_{11} = \frac{1}{m_q} + \frac{L_f^2}{I_o}, \quad b_{12} = \frac{1}{m_p} - \frac{L_f L_r}{I_o} \\
b_{31} = \frac{1}{m_b} - \frac{L_f L_r}{I_o}, \quad b_{32} = \frac{1}{m_b} + \frac{L_r^2}{I_o}
\]
It can be observed from (2.19) and (2.30) and the elements of the disturbance input matrix $G_{h_i}$ is not in phase with the actuator input matrix $B_{h_i}$, i.e. $\text{rank } [B_{h_i}] \neq \text{rank } [B_{h_i}, G_{h_i}]$, therefore the system does not satisfy the matching condition.

The following section presents the integration of the actuator dynamics to the active suspension dynamics. Therefore, the input signals to the active suspension systems now are the actual signals that control the actuators.

It can be observed that the non-zero elements of the disturbance matrix $G_{h_i}$ are not in phase with the input matrix $G_{h_i}$. Therefore the system suffer from the mismatched condition.

### 3.2 Road Profile

Road profile irregularities can be classified as being of smooth, rough minor and rough in natures. The smooth road profile represents the road disturbance with a single bump. The rough minor and rough in nature road profiles are represented by the uniform bumps height and non-uniform bumps height, respectively. These types of road profiles have been used by Yahaya [22] [23] [24] in his study in active suspension system.

The road disturbance $w(t)$ representing a single bump may be given by the following equation:

$$w(t) = \begin{cases} 
\alpha(1 - \cos \beta t), & t_1 \leq t \leq t_2 \\
0, & \text{otherwise}
\end{cases}$$

(3.32)
Where $a$ is the height of the bump $t_1$ and $t_2$ are the lower and upper time limit of the bump.

For this study the bumps height, $a$ used is 5 cm to simulate the road profile irregularities.

![Road Profile](image)

**Figure 3.2:** Road Profile.

### 3.4 Conclusion

From the model of the system, a transfer function needs to be derived to represent the system mathematically. This mathematical expression will use extensively through this research for many purposes such as analyzing transient response, determining stability and controllability.
CHAPTER 4

LINEAR QUADRATIC REGULATOR (LQR)

4.1 Introduction

A state space implement to linear model suspension system controller, the controller representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the differential and algebraic equations are written in matrix form. The state space representation (also known as ‘time domain approach’) provides a convenient and compact way to model and analyze the multiple inputs and multi outputs (MIMO). With $p$ inputs and $q$ outputs, we would otherwise have to write down $q \times p$. Laplace transforms to encode all the information about the system. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. ‘State space” refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that state space.

4.2 State Variable

The internal state variables are the smallest possible subset of system variables that can represent the entire state of the system at any given time. State variables must be linearly independent; a state variable cannot be a linear combination of other state variables. The minimum number of state variables
required to represent a given system, \( n \), is usually equal to the order of the system's defining differential equation. If the system is represented in transfer function form, the minimum number of state variables is equal to the transfer function's denominator after it has been reduced to a proper fraction. For example, in electronic systems, the number of state variables is the same as the number active elements in the circuit contributed by capacitors and inductors.

4.3 State Space Representation for Linear System

The most general state space representation of a linear system with \( p \) inputs, \( q \) outputs and \( n \) state variables is written in the following form:

\[
\dot{x} = A(t)x(t) + B(t)u(t)
\]  
\[y(t) = C(t)x(t) + D(t)u(t)
\]

where \( x = \) state vector, \( y = \) output vector, \( u = \) input (or control) vector, \( A = \) state matrix, \( B = \) input matrix, \( C = \) output matrix and \( D = \) feed through (or feed forward) matrix.

For simplicity, \( D \) is often chosen to be the zero matrixes, i.e. the system is chosen not to have direct feed through. Notice that in this general formulation all matrixes are supposed time-variant, i.e. some or all their elements can depend on time. The time variable \( t \) can be a "continuous" one (i.e. \( t \in \mathbb{R} \)) or a discrete one (i.e. \( t \in \mathbb{Z} \)): in the latter case the time variable is usually indicated as \( k \). Depending on the assumptions taken, the state-space model representation can assume the following forms:
Table 4.1: State space model representation

<table>
<thead>
<tr>
<th>SYSTEM TYPE</th>
<th>STATE-SPACE MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{x} = Ax(t) + Bu(t)$</td>
<td>Continuous Time Invariant</td>
</tr>
<tr>
<td>$y(t) = Cx(t) + Du(t)$</td>
<td></td>
</tr>
<tr>
<td>$\dot{x} = A(t)x(t) + B(t)u(t)$</td>
<td>Continuous time-variant</td>
</tr>
<tr>
<td>$y(t) = C(t)x(t) + D(t)u(t)$</td>
<td></td>
</tr>
<tr>
<td>$x(k + 1) = Ax(k) + Bu(k)$</td>
<td>Discrete time-invariant</td>
</tr>
<tr>
<td>$y(k) = Cx(k) + Du(k)$</td>
<td></td>
</tr>
<tr>
<td>$x(k + 1) = A(k)x(k) + B(k)u(k)$</td>
<td>Discrete time-variant</td>
</tr>
<tr>
<td>$y(k) = C(k)x(k) + D(k)u(k)$</td>
<td></td>
</tr>
</tbody>
</table>

4.4 State Space Controllability and Observability

Controllability is an important property of a control system, and the controllability property plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control. Controllability and observability are dual aspects of the same problem. Roughly, the concept of controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations. A continuous time-invariant state-space model is controllable if and only if,

$$\text{rank}[B \ AB \ A^2B \ \ldots \ A^{n-1}B] = n$$

(4.3)

The column rank of a matrix is the maximal number of linearly independent columns of the matrix. Likewise, the row rank is the maximal number of linearly independent rows of the matrix.
Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs. A system is said to be observable if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the output. A continuous time-invariant state-space model is observable if and only if,

\[
\text{rank} \begin{bmatrix} C \\
CA \\
\vdots \\
CA^{n-1} \end{bmatrix} = n
\]  

(4.4)

4.8 Controller Design Using Full State Feedback: Pole Placement

There is a various type of state space controller implementation such as pole placement, pole placement with reference input, Linear Quadratic Regulator (LQR), observer control, optimal control etc. The following explanation show the state space in most preferable for numerical computation, which is full state feedback controller (also called as pole placement controller). The schematic of a full-state feedback system is shown in Figure 4.1.

**Figure 4.1:** Full state feedback controller

From Figure 4.1, the plant is represented by the state space matrix as in the box (equation for \( \dot{x}(t) \) and \( y(t) \)). In typical feedback control system the output, \( y \), is feedback to the summing junction. It is now the topology of the design changes. Instead of feeding back \( y \), it wills then feedback all the state variables. If each state
variable is feedback to the control, \( u \), through a gain, \( K_i \), there would be \( n \) gain that could be adjusted to yield the required closed-loop pole values. The feedback through the gains \( K_i \) is represented in Figure 4.1 by feedback vector \( K \). The new state equations for the closed-loop system can be written as equation 4.5 and 4.6.

\[
x = Ax + Bu = Ax + B(\hat{K}x + r) = (A - BK)x + Br
\]

\[
y = Cx
\]

The characteristic equation of the closed-loop system can be written by inspection as,

\[
\text{det} [sI - (A - BK)] = 0
\]

4.6 The Optimal Control Theory

The starting point of an Optimal Control Theory is the set of state equations that describes the behaviour of dynamic system (plant) to be controlled. For a continuous-time system, the state equations are a set of first order differential equation, the state equations are a set of first order differential equation

\[
\dot{x}(t) = f(x, u, t), \quad t \in [t_0, t_1]
\]

(4.8)

Where \( x(t) \) is \( n \times 1 \) state vector, \( u(t) \) is \( p \times 1 \) input vector, \( f \) is a vector valued function and \( [t_0, t_1] \) is the control interval.

For discrete- time systems, the state equations are a set of first order difference equations
In this case, the system is a continuous-time system.

To design an Optimal Controller, there are several operational constraints that must be satisfied while achieving specified objectives. Following steps are involved in solution of an optimal control problem.

(i) For a given plant, find a control function \( u^* \) which will act upon the given plant in what is, in some known sense, the best possible way.

(ii) Realize the control function obtained from step (i) with help of a controller.

The design of an optimum controller is based on the following factors relating to the plant and to the nature of its connection with the controller:

(i) The characteristic of the plant.

(ii) The requirements made upon the plant

(iii) The nature information about the plant supplied to the controller.

4.7 Linear Quadratic Regulator (LQR) approach

The suspension system is the state regulator problem. Thus, the LQR approach is used in the designation of the system’s controller.

The objective is to transfer a system for initial state \( x(t) \equiv x^0 \) to the desired state \( x^1 \) (\( x^4 \) may in many cases be the equilibrium point of the system) with the minimum integral square error. Therefore, the LQR objective function is commonly called the Integral of Squared Error (ISE) criterion.
Relative to the desired state $x^1$, quantity $(x(t) - x^2)$ can be viewed as the instantaneous system error. If the system coordinates is transformed such that $x^1$ becomes the origin, then the new state $x(t)$ is itself the error.

Investigators have advanced the argument that the integral-square error

$$\begin{align*}
J &= \int_{t_0}^{t_1} \left[ \sum_{i=1}^{n_1} (x_i(t))^2 \right] dt \\
&= \int_{t_0}^{t_1} |x^T(t)x(t)| dt 
\end{align*}$$

is a reasonable measure of the system transient response from time $t_0$ to $t_1$. To be more general,

$$\begin{align*}
J &= \int_{t_0}^{t_1} \left( x^T(t)Qx(t) \right) dt 
\end{align*}$$

(4.10)

With Q a real, symmetric positive semi definite, constant matrix can be used as performance measure. The simplest form of Q one can used is a diagonal matrix:

$$Q = \begin{bmatrix}
q_1 & 0 \\
0 & q_2 \\
0 & q_3
\end{bmatrix}$$

(4.12)

The i-th entry of Q represents the amounts of weight the designer places on the constraint on the state variable $x_i(t)$. The larger the value of $q_i$ relative to the other value of $q$, the more control effort is spent to regulate $x_i(t)$

To minimize the deviation of final state $x(t_1)$ of the system from the desired state $x^1=0$, a possible performance measure is

$$J = x^T(t_1)Hx(t_1)$$

(4.13)

Where H is a positive semi definite, real, symmetric, constant matrix.

The design obtained by minimizing
may be unsatisfactory in practice. A more realistic solution to the problem is obtained if the performance index is modified by adding a penalty term for physical constraint on u. One of the ways of accomplishing this is to introduce the following quadratic control term in the performance index:

\[
J = \int_{t_0}^{t_1} (ux^T(t)Ru(t)) \, dt
\]

where R is a positive definite, real, symmetric, constant matrix. By giving sufficient weight to control terms, the amplitudes of control signals which minimize the overall performance index may be kept within practical bounds, although at the expense of increased error in \(x(t)\).

For the state regulator problem, a useful performance measure is therefore

\[
J = \frac{1}{2}x(t_1)Hx(t_1) + \frac{1}{2} \int_{t_0}^{t_1} (x^T(t)Qx(t) + u^T R u(t)) \, dt
\]

4.7.1 Infinite-Time State Regulator Problem

Since the suspension system has terminal time that is not considered \((t_f \rightarrow \infty)\), then the final state should be approach the equilibrium state \(x^1=0\) (assuming a stable system). So the terminal constraint in \(J\) is not necessary. For an infinite-time state regulator problem, the performance index is

\[
J = \frac{1}{2} \int_{t_0}^{t_1} (x^T(t)Qx(t) + u^T R u(t)) \, dt
\]

4.8 Conclusion
From the model of the system, a transfer function needs to be derived to represent the system mathematically. This mathematical expression will use extensively through this research for many purposes such as analyzing transient response, determining stability and controllability.
CHAPTER 5

SIMULATION

5.1 Introduction

The realization of suspension system was done by computational simulation in MATLAB program. The controller was firstly designed based on the mathematical modeling as discussed in Chapter 3. Once the optimum parameters of the controller were identified, it will then be interfaced with the SIMULINK block diagram. The SIMULINK will be designed by both m-files coding and SIMULINK model window in the MATLAB features. The designed SIMULINK supposes will perform with the LQR controller.

5.2 MATLAB Tool and Software

MATLAB, developed by MathWorks Inc., is a software package for high performance numerical computation and visualization. The combination of analysis capabilities, flexibility, reliability, and powerful graphics makes MATLAB the premier software package for electrical engineers.

MATLAB provides an interactive environment with hundreds of reliable and accurate built-in mathematical functions. These functions provide solutions to a broad range of mathematical problems including matrix algebra, complex arithmetic, linear systems,
differential equations, signal processing, optimization, nonlinear systems, and many other types of scientific computations. The most important feature of MATLAB is its programming capability, which is very easy to learn and to use, and which allows user-developed functions. It also allows access to FORTRAN algorithms and C codes by means of external interfaces. There are several optional toolboxes written for special applications such as signal processing, control systems design, system identification, statistics, neural networks, fuzzy logic, symbolic computations, and others.

MATLAB has been enhanced by the very powerful SIMULINK program. SIMULINK is a graphical mouse-driven program for the simulation of dynamic systems. SIMULINK enables students to simulate linear, as well as nonlinear, systems easily and efficiently.

The following section describes the use of MATLAB and is designed to give a quick familiarization with some of the commands and capabilities of MATLAB. For a description of all other commands, MATLAB functions, and many other useful features, the reader is referred to the MATLAB User's Guide.

5.3 Implementation of MATLAB in Suspension Control System

To design a controller for this system, we first need to derive the system dynamical equations. From these equations, we develop transfer functions and state space equation of the system. The concept of linearization is discussed here and is applied to the system to be controlled. Once the transfer function of the system is obtained, we design the controller that meets the performance requirements. Again, MATLAB is used as analysis and design tools. When a satisfactory compensator is obtained via simulation, implement their design using SIMULINK to allow the user to view the response plot.
5.4 Simulations

5.4.1 State Space

The half car model for passive and active suspensions has been obtained in Chapter 3. The mathematical model for passive suspension is given by equations (3.16-3.19). The mathematical model for the passive consists of the parameters for the springs and the dampers for the front and rear of the car body that are considered to be linear. This model assumed the road disturbance as input to the system. By substituting the parameters value of the suspension system as tabulated in Table 3.1 into equations (3.16-3.19), the passive suspension system for a half car model can be obtained as follows:

\[
\begin{bmatrix}
\dot{x}_b^f \\
\dot{x}_w^f \\
\dot{x}_b^r \\
\dot{x}_w^r
\end{bmatrix} =
\begin{bmatrix}
1.22 & 1.22 & 0.07 & -0.07 & -68.9 & 68.9 & 1.4 & -1.4 \\
13.7 & -13.7 & 0 & 0 & 767 & -1930 & 0 & 0 \\
0.09 & -0.09 & -1.41 & 1.41 & 5.02 & -5.02 & -26.2 & 26.2 \\
0 & 0 & 7.14 & -7.14 & 0 & 0 & 132.98 & -854.97
\end{bmatrix}
\begin{bmatrix}
x_b^f \\
x_w^f \\
x_b^r \\
x_w^r
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
W_f^f \\
W_f^r
\end{bmatrix}
\]

(5.1)
Equation (5.1) shows that the performance of the passive suspension system is purely affected by the road disturbance $w_f(t)$ and $w_r(t)$ for the front and rear wheels, respectively.

The mathematical model for the active suspension system with actuator dynamics for the half car model is given by equation (3.28-3.31). The model consists of springs, dampers and hydraulic actuators as discussed in Section 3.2. The parameter values used for the active suspension system are same as the passive value with additional $B$ matrix:

$$B = \begin{bmatrix} 1.026 \times 10^{-3} & -7.59 \times 10^{-5} \\ -11.47 \times 10^{-3} & 0 \\ -7.39 \times 10^{-5} & 1.407 \times 10^{-3} \\ 0 & -7.14 \times 10^{-3} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_f \\ f_r \end{bmatrix}$$  \hspace{2cm} (5.2)

This additional matrix is used to improve the performance of the half car suspension system in term of the ride comfort and road handling.

5.5.2 MATLAB Procedure for LQR Controller Design

**STEP 1:** Enter Matrix A and B:

$$A = \begin{bmatrix} -1.22 & 1.22 & 0.07 & -0.07 & -68.6 & 68.6 & 1.4 & 1.4 \\ 13.7 & -13.7 & 0 & 0 & 767 & -1930 & 0 & 0 \\ 0.09 & -0.09 & -1.41 & 1.41 & 5.02 & -5.02 & -26.2 & 26.2 \\ 0 & 0 & 7.14 & -7.14 & 0 & 0 & 132.98 & -854.97 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
```plaintext
B=[1.026e-3 -7.59e-5; -11.47e-3 0; -7.59e-5 1.407e-5; 0 0; 0 0; 0 0; 0 0]

C=[1 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0; 0 0 1 0 0 0 0 0; 0 0 0 1 0 0 0 0; 0 0 0 0 1 0 0 0; 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 1];

D=[0 0; 0 0; 0 0; 0 0; 0 0; 0 0; 0 0; 0 0];

STEP 2: Determine the controllability of open loop system:
sys = ss(A,B,C,D);
co = ctrb(sys);
ob = obsv(sys);
Controllability = rank(co)
Observability = rank(ob)

STEP 3: Obtain the matrices Q and R of the quadratic performance index:
x=100000;
Q=diag ([x x x x x x x])
x1=1e-3;
R = diag([x1 x1]);

STEP 4: Define the optimal feedback gain matrix, K:
```
\[ \begin{bmatrix} K, S, e \end{bmatrix} = \text{lqr}(A, B, Q, R) \]

**STEP4:** Run the m-file and get the value of K to be used in the SIMULINK model:

\[
K = 1.0 \times 10^4 \times \begin{bmatrix}
1.2720 & -0.8262 & -0.0546 & -0.0075 & 0.0729 & -6.9158 & -0.2376 & 0.5406 \\
-0.1219 & -0.0037 & -0.5164 & -0.9161 & -0.1403 & 0.4886 & 0.0029 & -0.1077
\end{bmatrix}
\]

Below are the descriptions of block diagram in the SIMULINK. These descriptions are intended to give the user a better understanding the function of each subsystem.

- **Road Profile** – The figure below shows the subsystem for the road profile. This road profile is used to present the road uncertainties for the half car suspension system.

  ![Figure 5.1: Road Profile](image)

- **Passive Suspension System** – The state space equations were transform line by line in order to simulate the passive response for the half car model. The F-function block is to put the state space value.
Figure 5.2: Passive Suspension System

Where,

$$Fcn = (-1.22*u[1]) + (1.22*u[2]) + (0.07*u[3]) + (-0.07*u[4]) + (-68.6*u[5]) + (68.6*u[6]) + (1.4*u[7]) + (-1.4*u[8])$$

$$Fcn1 = (13.7*u[1]) + (-13.7*u[2]) + (767*u[5]) + (-1930*u[6]) + (1160*u[9])$$

$$Fcn2 = (0.09*u[1]) + (-0.09*u[2]) + (-1.41*u[3]) + (1.41*u[4]) + (5.02*u[5]) + (-5.02*u[6]) + (-26.2*u[7]) + (26.2*u[8])$$

$$Fcn3 = (7.14*u[3]) + (-7.14*u[4]) + (132.98*u[7]) + (-854.97*u[8]) + (0.72*u[10])$$

c) **Active Suspension System** – This block consists of the passive block diagram with additional input which is the front and rear actuator input. This two input is used to control the system. The value of these two inputs obtained from the m-file that already discussed in the previous section. Because of the full state feedback is used for this system, all the output is selected and will be feedback to the system.
K1 and K2 is configured to select the rows inside the K matrix. The first row is used to control the front car, K1, and the second row to control the rear part of the car K2.
d) **Compare Passive and Active Response.**– The passive and active subsystem being put side by side and it also use the same road disturbance. The output of these two systems is overlapped by using the Mux block. This allow the passive and active response will be plotted in the same graph to make the analysis easier. An additional derivative block is needed for the body acceleration output for both front and rear car.

![Diagram of Passive and Active Suspension Systems](image)

**Figure 5.5:** Compare Passive and Active Response.

5.6 **Conclusion**

This chapter describes the development of the software controller in simulation by MATLAB program It also presents the construction of the SIMULINK to view the suspension system performance. This allows user to see the comparison between the active and passive response plot. The developed SIMULINK basically, consists of two main subsystems with the road disturbance being injected to simulate the actual car performance. The controllers need to be carefully tuned to get the best response.
6.1 Introduction

This chapter discusses the results of the open loop system without controller (passive system) for suspension system using half car model. Second, analysis the responses of the system with state-space controller for linear model are being applied. Finally, a comparative assessment of the impact of the LQR controller models on the system performance is presented and discussed.

To perform comparison between controller design for suspension control system, one of the first things that must be done during controller design is to decide upon a criterion for measuring how good a response is. For example, when we deal with systems where we are not bothered with the actual dynamics of how the steady state is reached, but only care about the steady state itself, a good measure will be the steady state error of the system defined by equation 6.1.

\[ e = x_{\text{final}} - x_{\text{ref}} \]  

However, in dynamic systems where the transient behavior is also important, it becomes important to introduce several other criterions. The most common are:
**Settling Time**: This is a measurement of the period of the system stabilize its new final value. It usually defined by the time it takes for the system response to come within a specified tolerance band of the set point value and stay there.

**Overshoot**: This is the maximum distance beyond the final value that the response reaches. It is usually expressed as a percentage of the change from the original value to the final, steady state value.

**Steady State Error**: This is a measure of how far the final value reached in the step response is from the actual desired value.

**Rise time**: It refers to the time required for a signal to change from a specified low value to a specified high value. Typically, these values are 10% and 90% of the step height.

### 6.2 Results for Half Car Suspension System

#### 6.2.1 Vertical Body Displacement

Vertical Body displacement is the difference between the car body, xb and the front and rear wheel, \(x_{wf}\) and \(x_{wr}\). The reduction in vertical body displacement means the increase in the car ride comfort.

For this system, there is only reduction of vertical body displacement for the rear part of the car. The slight increase in the front part of the car is still acceptable because it is within the spring range which is \(\pm 8\) cm for 5 cm road bump [24]. If it exceeds this range, it shows that the car spring is already not linear anymore. But both parts give much quicker response by increase in the settling time compared to the passive response.
The positive sign on the graph shows that the spring of the car is expanding while the negative sign shows the spring is shrinking. This reaction occurs because the spring tries to stabilize the car body.

![Front Vertical Body Displacement](image)

**Figure 6.1:** Front Vertical Body Displacement.
6.2.2 Vertical Wheel Displacement

This vertical wheel displacement graph is used to analyze the tire to road surface contact. The increase of overshoot in the graph shows that the car has better tire to road surface contact. The controller is designed to prevent the car from skidding or drifting when it hits the bump. Vertical wheel displacement is proportional to the road handling of the car.

Based on this study, the LQR controller cannot control both front and rear part at the same time. When the controller is applied, it only increases the rear car wheel to road surface contact. But the settling time for both parts increases a lot by using the LQR controller.
Figure 6.3: Front Vertical Wheel Displacement.

Figure 6.4: Rear Vertical Wheel Displacement.
6.2.3 Body Acceleration

Car body reaction also needs to be controlled to give better ride comfort and road handling. Body acceleration tells whether the system has good response in terms of the trajectories of the car. This body acceleration needs to be reduced.

Figure 6.5 and 6.6 shows that there are increases in body acceleration compared to the passive response. But again, the settling time for both responses improved after the LQR controller is applied.

**Figure 6.5:** Front Body Acceleration.
6.2.4 Force Input

The force input is similar to the controller input that being applied to the system. These force inputs are analyzed to check whether the Q and R value in LQR controller is realistic to actual life or not.

By looking at both graphs, the forces that being applied are acceptable to lift up a car and again the LQR controller try to give faster response for the system to settle.
Figure 6.7: Front Controller input.
6.2.5 System Stability

System stability can be analyzed by looking observing the eigenvalue of the system. The eigenvalue is the location of the poles in root locus. Passive system eigenvalues are shown below:

\[
\text{ans} = \begin{array}{l}
-7.2484 + 43.5780i, -7.2484 - 43.5780i, -0.2188 + 6.4031i, -0.2188 - 6.4031i, \\
-0.4921 + 4.6688i, -0.4921 - 4.6688i, -3.7757 + 28.9336i, -3.7757 - 28.9336i
\end{array}
\]

While the eigenvalues after the controller is applied are:

\[
\text{e} = \begin{array}{l}
-95.1679, -56.8740, -21.7785, -14.6414, -2.9167 + 5.5276i, \\
-2.9167 - 5.5276i, -1.2281 + 4.6099i, -1.2281 - 4.6099i
\end{array}
\]

In order to say the system is stable, the eigenvalue must satisfy these two conditions:

1. There must be no positive sign of eigenvalue- if there is one positive sign of eigenvalue, it shows that the pole is located at right hand plane of the root locus.
2. Eigenvalues with the controller are further than the open loop poles- if the poles located 5 to 10 time further than the system without controller, the system will become approximate to second order system.

From this studies, the system is stable and the controller is acceptable.
6.3 Conclusion

To perform comparison between the passive and controller design for suspension control system, one of the first things that must be done during controller design is deciding upon a criterion for measuring how good a response is. However, in dynamic systems where the transient behavior is also important, it becomes important to introduce several other criterions.

The most common are compare the percent overshoot $%OS$, peak time $T_P$, settling time $T_S$, rise time $T_R$ and percent steady state error, of the linear model using state-space controller to control the suspension control.
CHAPTER 7

CONCLUSION AND FUTURE WORK

7.1 Introduction

The project is considered as satisfactory where a half car model for a active suspension system based on LQR controller design. The modeling is implemented in simulation using MATLAB. A SIMULINK block diagram by using MATLAB command is also develops where it allow the user to view performance of suspension systems by comparing the passive and active response.

For control problem, it is important to get the desired rise time, peak time and settling time of the system. As a result, it is necessary to find out the best way to train the controller to get the desired response of the system. The stability of variety values of Q and R is then being compared to find out the optimum performance of suspension control system.

7.2 Conclusion

The project has progress from a hypothesis until accomplished of the desired objective. Throughout these two semesters, some critical problems have occurred especially in the logical aspect and implementation of SIMULINK. However, these problems were successfully solved and the project’s goal is achieve. The designed system is capable satisfying the design requirements.
Methodology to design a controller, which based on the Linear Quadratic Controller, also known as the optimal control has been presented. Besides, the difference of the three passive, semi-active and active suspension systems have mentioned. The mathematical derivations for the passive and active system have done in state space form with the road irregularities being injected to the system to simulate a real life situation response. These derivations are used to apply in the simulation using MATLAB software.

As a summary the controllers designed have clarify the expected simulation results and SIMULINK program also allow the user understand the simulation using computer software easily. The main purpose of the project has been achieved successfully.

However, the analysis results had shown that to achieve better simulation result, the controller need to be tuned carefully because each controller have their own tuning method. And there is a possibility where the overshoot of the system increase rapidly when the controller is applied.

A cruise control system is a system which requires a speed controller to maintain the speed at a desired speed. This can be achieved by reducing the error signal which is the difference between the output speed and the desired speed. There are various types of controller that can be use for that purpose.

7.3 Suggestion for Future Works

The future works are necessary to develop this project to a better stage that will more challenging and for used widely. Following are suggestion for future development:

1. Use the non-linear model.
Linear model neglect some parameter that will make a lot of difference in actual life. Non-linear model use different approach for the controller design and need to be study carefully.

2. Using higher level controller as a design tool:

   i) Variable structure control
   ii) Fuzzy logic controller
   iii) Neural network

   The different control technique can be used to design the controller for the system then simulation can be tested to obtain the comparison among all the controller.

3. Consider an attitude control of a complete car.

   As future work we might try to design a controller for the complete car system. This will achieve the better control performance for the vehicle such a body movement, suspension movement as well as the force disturbance.

4. Hardware design for suspension system.

   Hardware design need a very high budget to built the suspension plant. The hardware can be patent and commercialize because there are not much research on the hardware design.
REFERENCES


12. MATLAB Control System Toolbox.


20. SIMULINK™ User Guide.


APPENDIX A

MATLAB source code to determine value of K for LQR controller

A=[-1.22 1.22 0.07 -0.07 -68.6 68.6 1.4 -1.4; 13.7 -13.7 0 0 767 -1930 0 0; 0.09 -0.09 -1.41 1.41 5.02 -5.02 26.2 26.2; 0 0 7.14 -7.14 0 0 132.98 -854.97; 1 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0; 0 0 1 0 0 0 0 0; 0 0 0 1 0 0 0 0];

B=[1.026e-3 -7.59e-5; -11.47e-3 0; -7.59e-5 1.407e-5; 0 -7.14e-3; 0 0; 0 0; 0 0; 0 0];

C=[1 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0; 0 0 1 0 0 0 0 0; 0 0 0 1 0 0 0 0; 0 0 0 0 1 0 0 0; 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 1 0];

D=[0 0; 0 0; 0 0; 0 0; 0 0; 0 0; 0 0];

x=100000;
\( Q = \text{diag} ([x \ x \ x \ x \ x \ x \ x \ x \ x]) \)

\( x_1 = 1e-3; \)
\( R = \text{diag}([x_1 \ x_1]); \)

\([K,S,e] = \text{lqr}(A,B,Q,R)\)

\( \text{sys} = \text{ss}(A,B,C,D); \)
\( \text{co} = \text{ctrb(sys)}; \)
\( \text{ob} = \text{obsv(sys)}; \)

\( \text{Controllability} = \text{rank(co)} \)
\( \text{Observability} = \text{rank(ob)} \)
APPENDIX B

Presentation Slides for Project

Content

1. Introduction
2. Objective
3. Problem Statement
4. Literature Review
5. Methodology
6. Mathematical Modeling
7. Controller Design - LQR
8. Simulation & Results
9. Discussion and Future Suggestion
10. References
1.0 INTRODUCTION

- A suspension system may be categorized as passive, semi-active or active suspension system.
- Good ride comfort as well as road handling can be improved by electronically controlled suspension system.
- Active suspension system is considered to increase the ride quality.

Active Suspension System

- Has an ability to store, dissipate and to introduce energy to the system.
- The hydraulic actuator is connected in parallel with the spring and absorber.
- Sensors are located at different points of the vehicle to measure the motions of the body.
- It may vary its parameters depending upon operating condition.
2.0 OBJECTIVE

- To analyze and understand active suspension system for half car.
- To study the different between quarter car and half car model.
- To construct mathematical model for the system.
- To design a suitable controllers design to solve active suspension system.
3.0 PROBLEM STATEMENT

- CONVENTIONAL SUSPENSION - Cannot give comfort ride to a different road surface quality and driving situation.

- Active suspension is proposed to improve the riding comfort, unfortunately the controller need to be design carefully in order to get a good performance due to complexity of the model.

4.0 LITERATURE REVIEW

Past Undergraduate Studies Within UTM

<table>
<thead>
<tr>
<th>PERSON</th>
<th>MODEL</th>
<th>FINDINGS</th>
</tr>
</thead>
</table>
| Khoo, Teng Hong (2000) | Quarter Car Model | Controller- Pole Placement & LQR controller  
Result- The simulation result shows that LQR is the best way to increase the ride comfort & road handling |
| Yam, Su-Anne(2000)  | Quarter Car Model | Controller- PID & LQR Controller  
Result- Graphical User Interface (GUI) was develop to simulate the result of the quarter |
## Past Research on Active Suspension System

<table>
<thead>
<tr>
<th>Person</th>
<th>Findings</th>
<th>Controller</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yahaya Md. Sam, Mohd. Ruddin Hj. Ab. Ghani, Nasarudin Ahmad (2001)</td>
<td>Controller: LQR controller &lt;br&gt;Result: An active suspension also increase a tire to road contact in order to make the vehicle more stable.</td>
<td>LQR</td>
<td></td>
</tr>
<tr>
<td>Joshua Vaughan, William Singhose, and Nader Sadegh (2003)</td>
<td>Controller: LQR&lt;br&gt;Result: The addition of payloads can greatly affect the dynamic response of a vehicle.</td>
<td>LQR</td>
<td></td>
</tr>
<tr>
<td>Atonin stribrsky, katerina hyniova, Jaroslac Honchu (2003)</td>
<td>Controller: Fuzzy Logic Controller &lt;br&gt;Result: active suspension control system is proposed to achieve both ride comfort and good handling. This aim was achieved with respect to the simulation results.</td>
<td>Fuzzy Logic</td>
<td></td>
</tr>
</tbody>
</table>

## 5.0 Methodology

1. **Understand the title, scope and objective**
2. **Define project**
3. **Get information (Quarter car & Half Car)**
4. **Do the simulation using Matlab**
5. **Design LQR controller**
6. **Decide software (Matlab)**
7. **Compare result from with various road profile**
6.0 Mathematical Modeling

Figure 2: Half Car Suspension system Model.

- Where
  - $m_b$: mass of the car body (kg).
  - $I_b$: moment of inertia for the car body (kgm$^2$).
  - $\theta_b$: rotary angle of the car body at the center of gravity (rad).
  - $m_{wf}$: mass of the front wheel (kg).
  - $m_{wr}$: mass of the rear wheel (kg).
  - $k_{sf}$: stiffness of the front car body spring (N/m).
  - $k_{bf}$: stiffness of the rear car body spring (N/m).
  - $k_{wr}$: stiffness of the front car tire (N/m).
  - $k_{wf}$: stiffness of the rear car tire (N/m).
  - $c_{sf}$: damping of the front car damper (Ns/m).
  - $c_{bf}$: damping of the rear car damper (Ns/m).
  - $x_b$: vertical displacement of the car body at the center of gravity (m).
  - $x_{bf}$: vertical displacement of the car body at the front location (m).
  - $x_{wf}$: vertical displacement of the car body at the rear location (m).
  - $x_{sr}$: vertical displacement of the car wheel at the front wheel (m).
  - $x_{wr}$: vertical displacement of the car wheel at the rear wheel (m).
  - $w_f$: an irregular excitation from the road surface at the front car (m).
• $w_r$ = an irregular excitation from the road surface at the rear car (m).
• $L_f$ = distance of the front suspension location with reference to the center of gravity of the car body (m).
• $L_r$ = distance of the rear suspension location with reference to the center of gravity of the car body (m).
• $L$ = total distance of the front suspension to the rear suspension, i.e. $L_f + L_r$ (m).

The motion equation for the passive suspension for the half car model may be derived as follows:

$$m_b \ddot{x}_b + c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf}(x_{bf} - x_{wf}) + c_{br}(\dot{x}_{br} - \dot{x}_{wr}) + k_{br}(x_{br} - x_{wr}) = 0 \quad (2.1)$$

$$I_b \ddot{\theta}_b + L_f[c_{bf}(\dot{x}_{bf} - \dot{x}_{wf})] + k_{bf}(x_{bf} - x_{wf}) - L_r[c_{br}(\dot{x}_{br} - \dot{x}_{wr})] + k_{br}(x_{br} - x_{wr}) = 0 \quad (2.2)$$

$$m_{wf} \ddot{x}_{wf} - c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) - k_{bf}(x_{bf} - x_{wf}) + k_{wf}(x_{wf} - w_f) = 0 \quad (2.2)$$

$$m_{wr} \ddot{x}_{wr} - c_{br}(\dot{x}_{br} - \dot{x}_{wr}) - k_{br}(x_{br} - x_{wr}) + k_{wr}(x_{wr} - w_r) = 0 \quad (2.4)$$
Hence, the state space representation of the active suspension of the half car model may be obtained as:

\[
\begin{align*}
\dot{x}_{h_{la}}(t) &= A_{h_{la}}x_{h_{la}}(t) + B_{h_{la}}u_{h_{la}}(t) + G_{h_{la}}w_{h_{la}}(t) \\
\end{align*}
\]

Where,

\[
\begin{align*}
x_h(t) &= [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T \\
\end{align*}
\]

\[
\begin{align*}
\dot{x}_h(t) &= [\dot{x}_b, \dot{x}_w, \dot{x}_br, \dot{x}_w, x_b, x_w, x_br, x_w]^T \\
\end{align*}
\]
Modeling - Road Profile

The road disturbance $w(t)$ representing a single bump may be given by the following equation:

$$
w(t) = \begin{cases} 
a(1 - \cos 8\pi t), & t_1 \leq t \leq t_2 \\
0, & \text{otherwise} \end{cases}$$
7.0 Controller Design - LQR

- Linear Quadratic Control $\rightarrow$ Optimal control

Design step in LQR
1. Determine the controllability of open loop system.
2. Find the controller matrix gain, $K$ using "lqr" function in MATLAB.
   - Choose two parameter $R$ and $Q$ which will balance the relative of input and state in cost function that we are trying to optimize.
   - Set $R$ to $1e{-3} \rightarrow R = \text{diag}([1e{-3} \quad 1e{-3}])$
   - Set $Q=C^T C \rightarrow x=100000; \quad Q=\text{diag}([x \times x \times x \times x \times x])$
3. The value of $K$ obtain after run the simulation are:

$$K = 1.0e+004^*$$

$$\begin{bmatrix}
1.2720 & -0.8262 & -0.0546 & -0.0075 & 0.0729 & -6.9158 & -0.2376 & 0.5406 \\
-0.1219 & -0.0037 & -0.5164 & -0.9161 & -0.1403 & 0.4886 & 0.0029 & -0.1077 \\
\end{bmatrix}$$
8.0 Simulation and Result

Figure 3: SIMULINK Block Diagram

Figure 4: Inside the Suspension Block
Figure 5: Front Vertical Body Displacement

Figure 6: Front Vertical Wheel Displacement
Figure 7: Body Velocity

Figure 8: Body Acceleration
9.0 Discussion

- Even though the percent overshoot slightly bigger than the passive response, it is still under comfort limit for suspension system which is ±8cm for 5cm bump.
- The Settling time is also improved by using the LQR controller.
- From this studies, we can conclude that LQR can be used as a method to increase the ride comfort and road handling to passengers.

10.0 Future suggestion

- Use full car model
- Implement intelligent controller:
  - Fuzzy logic
  - Neural network
  - Neural fuzzy
- Include the hydraulic model in the system.
- Develop Graphical User Interface (GUI) - animation.
- Design hardware for Active Car Suspension.
11.0 References


[16] "How Does Active Suspension Work?" http://www.bath.ac.uk/ -en7prw/how.htm
Q & A

THANK YOU